
Harnessing Probability for Dynamic and Balanced Game Design

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Abstract.

Probability distribution frameworks are used in gaming to model random events and outcomes. The paper explores the fascinating intersection of mathematics and game design, focusing on the pivotal role of probability distributions in shaping dynamic game play experiences. It delves into how randomness, uncertainty, and fairness: key elements of great games, are meticulously modelled using mathematical tools. From the uniform distribution governing dice rolls to complex poisson processes driving procedural generation, the paper focuses on various statistical models highlighting wide range of mechanics across genres. With case studies, real-world examples, and step-by-step explanations, the paper act as a guideline for a person interested in the mathematical foundations and applications of probability distribution models in game design.

Keywords probability distribution models, game play, bernoulli distribution, binomial distribution, poisson distribution, normal distribution, uniform distribution, exponential distribution

1. INTRODUCTION

Probability distributions play a pivotal role in the design, mechanics, and evaluation of games. In game design, probability distributions are used to determine the likelihood of events, such as dice rolls, card draws, or loot drops, ensuring fairness, unpredictability, and balance [1, 2]. Distributions like uniform, normal, binomial, and poisson has been applied to control the frequency and impact of in-game events, such as damage outputs, spawning of enemies, or resource generation. Moreover, they influence strategic decision-making, where players assess risks and rewards based on probabilistic outcomes. From determining the outcome of a dice roll in board games to controlling enemy spawn rates in video games, probability distributions govern the likelihood of events, shaping player experiences and decision-making. Understanding the types and functions of probability distributions such as uniform, normal, and poisson distributions- enables developers to craft systems that enhance fairness, excitement, and immersion. Whether simulating realistic physics, creating procedurally generated worlds, or designing adaptive AI behaviors, probability distributions play a vital role in delivering rich and dynamic game play experiences [3, 4].

2. RELATED WORK

Recent studies have explored how probability is applied to create branching narratives where player choices lead to different story outcomes. Research work in [5] looks at how probabilistic decision-making algorithms influence the pacing, character development, and narrative branching in story-driven games. Research in [6] looks at how probability distributions has been used to scale game difficulty dynamically. By applying models like poisson or gaussian distributions, game designers can adjust the likelihood of challenging events occurring, offering a more personalized player experience based on past performance. Studies like those of [7] demonstrate how game designers incorporate probabilistic models into player behavior and skill-based decisions. Also, work in [8] discuss how randomness through the use of normal and binomial distributions in role-playing games has added variability to game play, making player experiences less predictable and more engaging.

3. METHODOLOGY

Probability distributions contribute to the gaming experience by introducing key elements like unpredictability, fairness, strategy, emotional highs and lows, dynamic difficulty, and replayability. Through their controlled randomness, they allow developers to create engaging, balanced systems that encourage decision-making, challenge players, and keep them invested in the outcome of each random event.

In simple games, the **Bernoulli Distribution** has been used to model the events where the outcome of a single event has been either be a success or a failure, such as flipping a coin, rolling a die for a specific outcome, or

determining if an action in the game is successful or not. Below is an example of how the **Bernoulli Distribution** has been applied in simple games. In a simple archery game, the player has one shot to hit a target. Here, the Bernoulli distribution is used to model the outcome of the players shot, where the event of hitting the target is a success, and missing the target is a failure. Formula for Bernoulli Distribution: $P(X=1)=p=0.7$ (for hit, success) . $P(X=0)=1-p=0.3$ (for miss, failure).

Moving on next is the description on **Binomial Distribution** which is one step ahead of Bernoulli trials. The binomial distribution is particularly useful when there are multiple independent trials, and each trial has the same probability of success. Imagine a simple game where a player takes a series of shots in basketball. Each shot has a chance of being successful, and the player wants to know how many successful shots they will make after a series of attempts. **Question: what is the probability that the player will make exactly 7 shots out of 10?**

Applying the Binomial Distribution: We can model this situation using the binomial distribution where: $n=10$ (number of shots), $p=0.7$ (probability of making a shot), $k=7$ (number of successful shots). The binomial probability formula is:

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$P(X=7) = \binom{10}{7} (0.7)^7 \times (0.3)^3 \approx 0.267$$

So, the probability of making exactly successful shots out of 10 is about 26.7%.

When we proceed with the next discrete probability distribution, it is **Poisson Distribution** which is commonly used in games to model the occurrence of **rare events** over a fixed interval of time or space. In games, the Poisson distribution has been applied to model random occurrences that are relatively rare but impactful, such as enemy spawns, random loot drops, or other events that do not happen continuously but can occur at unpredictable intervals. The Poisson distribution can be used to model the frequency of these events, particularly when the loot drops are rare but occur at a known average rate.

Example:

In a **loot box** system or after player defeats enemies in a game, items might drop according to a Poisson distribution. For instance, assume that on an average, 2 items drop per 10 enemies defeated.

- $\lambda=2$: The average number of items dropped per 10 enemies.
- The probability of getting exactly **3 items** after defeating 10 enemies is calculated using the poisson formula:

$$\text{for } k=3; P(X=3) = \frac{2^3 \times e^{-2}}{3!} = \frac{8 \times e^{-2}}{36} \approx 0.1804$$

So, the probability of getting exactly 3 items from 10 enemies defeated is about **18.04%**. This helps create games where players experience randomness in rewards, with the system ensuring that loot is distributed unpredictably but in a controlled manner over time.

Next is the discussion on continuous distribution. The **Uniform Probability Distribution** is one of the simplest and most commonly used continuous distributions in games. In a uniform distribution, all outcomes are equally likely, but each one has the same probability of happening making it an essential tool for modeling random events in a fair and predictable manner. The simplest and most iconic examples of a uniform distribution in games are the use of dice rolls in board games like *Monopoly and Snakes and Ladders*. A standard six-sided die has a uniform distribution, where each face (numbered 1 to 6) has an equal probability of **Each Outcome**. A fair six-sided die has six possible outcomes: {1,2,3,4,5,6} and each face has an equal probability of appearing, $P(\text{Outcome}) = \frac{1}{6} \approx 0.1667$ (16.67%)

In many games, especially those that involve dice rolls, cards, or other random elements, the outcomes can often be modelled by another kind of continuous distribution known as **Normal Distribution**. The normal probability distribution in games is primarily used to understand and predict the likelihood of different outcomes, assess risk, optimize strategies, and make better decisions based on probabilistic thinking. When the events occur repeatedly (over multiple rounds or trials), their aggregate distribution often approximates a normal distribution.

- **Example of Card Shuffling in a Game:** Imagine a deck where:

- **Low-value cards** are 70% of the deck. **Mid-value cards** make up 20% of the deck.
- **High-value cards (e.g., legendary or powerful cards)** make up 10% of the deck.

In summary, the **normal distribution** in card games helps model how random events (like drawing cards) are likely to result in outcomes that are centered on the "average" case, with extreme cases being rare but still possible. This makes the game both **exciting** and **balanced**, where skill and strategy can still shine through despite the inherent randomness.

The last part of the discussion is on another continuous distribution known as **Exponential Distribution**. The **exponential distribution** is commonly used in games, particularly for modelling random events that happens continuously and independently over time. It's most common use in games is to determine the time between events, such as spawn times, resource replenishment, or triggering random encounters.

▪ **Example: Spawn Timing for Enemies in a Game**

Objective: Ensure that on average, 1 enemy spawns every 15 seconds, but the actual time between spawns varies randomly. The mean time between spawns is 15 seconds.

The rate parameter (λ) is, $\lambda = \frac{1}{\text{mean time}} = \frac{1}{15} \approx 0.067$ (spawns per second)

4. EXPERIMENTAL RESULTS

4.2.1 **Scenario1:** Below is a simple example in which we use Bernoulli distribution in a game scenario, where a player has a chance to win or lose based on a specified probability. The player has a 60% chance to win each round. The visualization graph is similar to the one as follows:

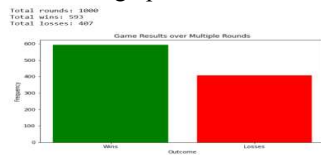


Figure4.6. Bernoulli Distribution of a Game with multiple Rounds

Scenario2: Next, is the binomial distribution which is a probability distribution that has been used to describe **the number of successful or unsuccessful outcomes in a series of events**. To find the probability of getting an even number of heads when flipping 21 coins, we need to calculate the probability of getting 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, or 20 heads. The visualization graph is shown as follows:

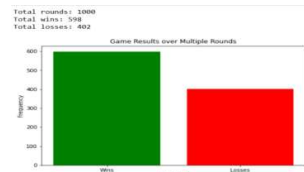


Figure4.2. Binomial Distribution of Flipping 21 Coins

Scenario3: In this example, we have simulated drawing cards from a deck where **rare cards** appear randomly, following a **Poisson distribution**. We have assumed that, on an average, 1 rare card drops every 10 card draws.. The visualization graph is similar as follows:

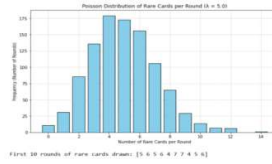


Figure4.3. Poisson Distribution of rare drawn after a certain number of draws

Scenario4: Consider a **sports simulation game** example where we model **player performance** using **Normal Distribution**. In this scenario, we have simulated a player's score (such as points in a basketball game or goals in a soccer match) where most of the time the player scores near the average. The visualization graph is similar as follows:

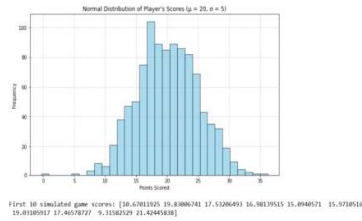


Figure4.4 Normal Distribution of Player Performance with variability around average

Scenario5: Below is a simple example of **Uniform distribution** used for **random player selection** in a team scenario. Imagine we have a soccer team with 5 players. We want to randomly select a player for a task, like taking a penalty shot. Every player has an equal chance of being chosen. The visualization graph is as follows.

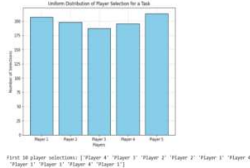


Figure4.5 Uniform Distribution of Player Selection for a soccer team

Scenario6: In a basketball game, time between fouls can be modeled using an Exponential distribution. We have simulated the time between fouls in a game and calculated when each foul occurs during a 48-minute basketball game. The visualization graph is similar as follows:

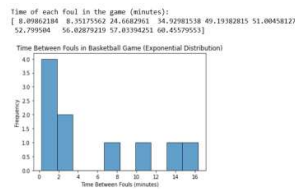


Figure4.6 Exponential Distribution for time between Fouls in basket ball game'

5. CONCLUSION

Probability distributions are a key component in modern game design, offering developers the ability to shape dynamic and engaging experiences. Careful implementation and testing ensure that randomness contributes positively to the overall game experience rather than detracting from it.

6. REFERENCES

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